Ex. 3. Exchange Banco Marks 5486 12 Sch. at HAMBURG, with

PARIS, at 187¾

If 100 Mks — 187¾ Frs
— Marks 5486 12 ?

GENOA, at 187¼

If 100 Mks — 187¼ Lire
— Marks 5486 12 ?

AMSTERDAM, at 35¼

If 40 Mks — 35¼ Flor.
— Marks 5486 12 ?

LEGHORN, at 43¼

If 43¼ Schill. — 1 Pezza
— Marks 5486 12 ?

FRANKFORT, at 148

If 300 Mks — 148 R. D.
— Marks 5486 12 ?

BERLIN, at 153

If 300 Mks — 153 Pruss. D.
— Marks 5486 12 ?

VIENNA, at 147

If 200 Mks — 147 Flor.
— Marks 5486 12 ?

PETERSBURG, at 9¼

If 9¼ Schill. — 1 Ruble
— Marks 5486 12 ?

Ex. 4. Exchange Rixd. 8128 72 Kr. W. Z. at FRANKFORT, with

PARIS, at 78¾

If 78¾ R. D. — 300 Frs
— Rixd. 8128 72 ?

BREMEN, at 109¾

If 109¾ R. D. — 100 R. D.
— R. D. 8128 72 ?

AMSTERDAM, at 139½

If 139½ R. D. — 250 Flor.
— Rixd. 8128 72 ?

HAMBURG, at 146

If 146 R. D. — 300 Mks
— Rixd. 8128 72 ?

AUGSBURG, at 100¾

If 100¾ R. D. — 150 Flor.
— Rixd. 8128 72 ?

BERLIN, at 104½ Kr.

If 1 R. D. — 90 Kr. W. Z.
46 K. W. Z. — 55 K. 24 GF.
104½ K. 24 GF. — 1 Pruss. D.
— Rixd. 8128 72 W. Z ?

Ex. 5. Exchange 8000 Lire Nuove at GENOA, with

PARIS, at 99	VIENNA, at 252
If 99 Lire — 100 Francs — Lire 8000 ?	If 252 Lire — 100 Florins — Lire 8000 ?
AMSTERDAM, at 207	LEGHORN, at 512
If 207 Lire — 100 Florins — Lire 8000 ?	If 512 Lire — 100 Pezze — Lire 8000 ?
HAMBURG, at 182	LISBON, at 480
If 182 Lire — 100 Marks — Lire 8000 ?	If 480 Lire — 100 Milreis — Lire 8000 ?

PRODUCTS.

Ex. 1.	Flor.	6761 63 Cts.	Flor.	5601 55 Kr.
	Mks	7650 10 Sch.	Lire	14067 33 Cent.
	R. D.	3742 6 Kr.	Pezze	2778 19 s 9 d.
	Rub.	13184 04 Cop.	Lire	16772 59 Cent.

Ex. 2.	Frs.	9997 21 Cts.	Ducats	2384 75 Grani
	R. D.	2613 38 Kr.	Lire	10094 18 Cent.
	Mks.	5374 1 Sch.	Pezze	1966 16 s 1 d.
	Flor.	3947 10 Kr.	Reis	2:062.486

Ex. 3.	Frs.	10301 37 Cts.	Lire	10273 94 Cent.
	Flor.	4835 20 Cts.	Pezze	2029 15 s 7 d.
	R. D.	2706 72 Kr.	Pruss. D.	2798 7 Gros.
	Flor.	4032 46 Kr.	Rubles	9490 59 Cop.

Ex. 4.	Frs.	30966 86 Cts.	R. D.	7406 47 Grotes.
	Flor.	14567 74 Cts.	Mks	16703 0 Schill.
	Flor.	12102 26 Kr.	Pruss. D.	8370 19 Gros.

Ex. 5.	Frs.	8080 81 Cts.	Flor.	3174 36 Kr.
	Flor.	3864 73 Cts.	Pezze	1562 10 Soldi
	Mks	4395 10 Sch.	Reis	1:666.667

INDIRECT EXCHANGES,

OR

ARBITRATIONS OF EXCHANGE.

Indirect Exchanges are operations of exchange between two countries, through the medium either of one other country, or of several other countries, and the proportional rates of exchange deduced from them, are called Arbitrated Rates of Exchange.

Arbitrations of Exchange, or the calculations of arbitrated rates, are therefore either Simple or Compound.

SIMPLE ARBITRATIONS.

The Rate of Exchange produced by drafts or remittances between two places, is called a Simple Arbitrated Rate, when there is only one intermediate place employed in the operation ; thus the rate of Exchange between London and Paris, obtained by the purchase in London of Bills upon Amsterdam, and by the sale of these Bills in Paris, is called a Simple Arbitrated Rate.

The object of the calculation of an Arbitrated Rate of Exchange, is to find whether it will produce a better rate, for either remitting or drawing, than is afforded by the direct rate ; and the object of the calculations of several rates, is to find which is the best for either of the same purposes.

In making these comparisons, in order to guide our operations, it is necessary for us to inquire,

1st—Whether the operation is, on the one hand, to be a remittance, or, on the other, to be a draft or return.

2ndly—Whether the variable price, or the rate of exchange, is in Foreign money or in Sterling.

Then we have to draw the following conclusions:

For remittances,

With a Foreign rate, any arbitrated rate is better than the direct rate, if it is greater than the direct rate.

With a Sterling rate, any arbitrated rate is better than the direct rate, if it is less than the direct rate.

Because, in either case, a given sum in Sterling will produce a greater sum in Foreign money, or a given sum in Foreign money will cost a less sum in Sterling.

For drafts or returns,

With a Foreign rate, any arbitrated rate is better which is less than the direct rate.

With a Sterling rate, any arbitrated rate is better which is greater than the direct rate.

Because we shall obtain, in either case, a greater sum in Sterling from a given sum that we draw for in Foreign money.

With inquiries of this nature is generally connected, a comparison between the direct rates of the two places concerned, for the purpose of determining, for either remittances or drafts, which place should make the operation.—Thus for remittances to Paris, or to create a fund in Paris, the direct rates are compared to determine whether we, in London, should remit to Paris, or whether our correspondents in Paris should draw upon us; and of course the reverse, if the remittance is to be made from Paris, or the fund is to be created in London.—In either case, and according to whether the variable price is in Foreign

money or Sterling, we must be guided by similar considerations to the preceding; namely, for remittances abroad to prefer that mode which gives the greatest sum in Foreign money, or costs the least sum in Sterling; and for returns or remittances to this Country, to prefer that which costs the least Foreign money, or yields the greatest sum in Sterling.

For either direct operations, or for comparisons with the arbitrated rates, the direct rates must be discounted or reduced to Short Prices,* because, for the former purposes, the interest for the time which the Bill has to run affects the price,† operating contrary ways upon the rates in the two countries, and, for the latter, the arbitrated rates are necessarily calculated for present money, as Bills are never sold on what may be called credit: and it is here necessary also to observe, that in making remittances of indirect Paper upon speculation, Bills at a long date are preferable to those at a short date, because more time is afforded, if it should be judged proper to wait for an improvement in the rate of Exchange.

Bills at short sight are rarely ever used in operations of this kind.

In order, therefore, to admit of a proper comparison between the different sorts of Paper, the direct rates also at the two places of operation should be taken at a long date, as 3 months, and then discounted, either according to the rate charged by the two houses of business, or at the market rate of discount.

To show how to apply this discount properly, we will take the rates in a following example, page 95.

London on Paris at 3 months is quoted   Fr. 25 55 Cents.
Paris on London at 3 months   —   Fr. 25 10

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* Short prices are the prices of Bills at sight, or at short sight, which is generally 3 days' sight.

† Any small differences is not in this country taken into consideration; thus a Bill at 90 days' date generally obtains as good a price as a Bill at 75 days' date; however, under equal circumstances in other respects, the latter Bill would obtain a preference for remittances abroad, because it would there usually sell for something more than the Bill at the longer date.

The discount for 3 months is there stated to be taken at 1 per Cent, or 25 Cents,* which is taken from the London rate, and added to the Paris rate, to make them Short or Cash rates; rendering the one Frs. 25 30 Cents, and the other Frs. 25 35 Cents.

The Interest is taken from the London rate, because if I send the Bill to Paris, and get it discounted there, the Interest will be deducted; but it is added to the Paris rate, because, if at Paris I want a Bill upon London at sight, I shall have more French money to pay for it, than I should have to pay for a Bill at 3 months.

It is evident, that if the variable price is in Sterling, as the rate with Madrid, the allowance for Interest must be reversed, and must be added to the London price, and subtracted from the price abroad.

It now remains to exhibit the usual method of stating and working by the Chain Rule, or Rule of Equations, the arithmetical Questions of arbitrated rates.—The requisite data in these operations, are, 1st, The fixed price of the rate between the two places, of which fixed price the arbitrated variable price is required. 2ndly, The buying price at the place where the operation commences. 3rdly, The selling price at the other place concerned ; and, occasionally, there are intermediate or auxiliary rates required, to connect the prices, either with each other or with the fixed price, or to bring out the result in the proper money of the variable price.

The contractions that may be practised, and the method of finding and employing fixed numbers, we shall show in the following calculation.

---

* To be very accurate, the time which the post takes between the two places should be deducted from the time which the Bill has to run.

The Interest is here reckoned at 4 per Cent per Annum, but perhaps the more usual rate is 5 per Cent per Annum ; and it is to be noticed, that although the Interest for the difference of the time is, strictly speaking, the proper difference in the rates, as far as they are made subjects of calculation, yet in the actual negociation of Bills the difference is generally less, because Bills at sight are usually in less demand than Bills at a long date.

EXAMPLE

Of the arrangement of the terms of an Equation for finding an Arbitrated Rate.

To find the Arbitrated Rate of Exchange between London and Frankfort, or the number of Batzen per £ Sterling, which may be produced by Bills upon Berlin, bought in London at the rate of 6 Pruss. Dollars 25 Silver Groschen per £ Sterling, and sold in Frankfort at 104 Kreuzers in 24 Guldenfuss.

We have here,

1st —The fixed Price,   viz. 1   £ Sterling.
2ndly—The buying Price, — 6   D. 25 G. per £ Ster.
3rdly—The selling Price, — 104 Kr. per Dollar.

but as the rate between London and Frankfort is expressed in Batzen, and the selling price is in Kreuzers, we have to add also this rate,

4 Kreuzers   are   1 Batz ;

and, further, as the rate with London is in Wechselzahlung, while the selling price is in 24 G. F. we must add this rate,

55 Batzen in 24 G. F.   are   46 Batzen in W. Z.

We then have to arrange these data, so that the terms may lead from one to the other, and that the first may be the fixed price, and the last the money in which it is required to find the arbitrated value, thus ;

$$
\begin{array}{lllll}
 & & & 1 & £ \text{ Sterling ?} \\
£ 1 & — * & 6 & 25 & \text{Doll. and Gr.} = 205 \text{ Gros.} \\
30 \text{ Gr.} = \text{Doll. } 1 & — & * & 104 & \text{Kreuzers.} \\
\text{Kreuz. } 4 & — & & 1 & \text{Batz in 24 G. F.} \\
\text{in 24 G. F. } 55 & — & & 46 & \text{Batzen in W. Z.}
\end{array}
$$

Having stated the terms of the equation, we have here reduced the Prussian Dollars into Groschen, at 30 to the Dollar, but we might have retained the Dollars, and reckoned 6 Doll. 25 Gr. as $6\frac{25}{30}$ or $6\frac{5}{6}$ Dollars, without reducing the 1 Dollar to Groschen.

The whole being thus arranged and prepared for calculation, we may find the result as usual, by multiplying into one product the consequents, or right hand terms, and dividing it by the product of a similar multiplication of the antecedents, or left hand terms; or we may divide any two terms on opposite sides, by any number that will exactly divide them, and use the quotients in their stead, which may be done either in the equation, or in the following fractional form of arranging the numbers operated with.

$$\frac{46 \times \overset{26}{104} \times \overset{41}{205}}{\underset{11}{55} \times 4 \times 30} = \frac{49036}{330} = 148.593$$

We here divide 104 and 4 by 4, and 205 and 55 by 5, and the operation is then reduced to the multiplication of 46 by 26 and by 41, and the division of the product by 11 times 30, or 330, as

$$
\begin{array}{r}
46 \\
26 \\
\hline
1196 \\
41 \\
\hline
330) 49036 \\
\hline
\end{array}
$$

Result   148.593 &c. Batzen.

It must be observed, that when the Equation is used for arbitrated rates, instead of a general reduction of the terms, those which are fixed or invariable are usually compounded into a fixed number, and the variable terms or rates are applied to it, as multipliers or divisors, according to which side of the Equation they belong.

Thus taking from the preceding Equation the invariable terms, and compounding them in the following manner, we obtain the following fixed number :

$$\begin{array}{c} 55 \\ 120 = 4 \ \times \ 30 \\ \hline 6600 \ ) \ 46.000 \end{array}$$

Fixed Number   0.0069697 nearly.

Upon this fixed number the variable numbers of the two rates are used as multipliers, both being upon the right-hand side of the equation, thus ;

Fixed Number   0.0069697 * for Grotschen
205
                ———————
                348485
                139394
                ———————
                1.4287885
                104
                ———————
Batzen   148.594   Result

This form may be varied by not reducing the Dollars, then we have

$$\begin{array}{c} 55 \\ 4 \\ \hline 220 \ ) \ 46.0 \end{array}$$

Fixed Number $0.2\frac{1}{11}$ for Dollars.

---

* The variable terms are, in the preceding, and also in every following, Equation, marked with an asterisk. It is not the common practice to give a decimal form to fixed numbers for finding arbitrated rates, because the number of places of whole numbers is always known, therefore the above would generally be given 69697 without any valuation.

And on account of its form this number is best used as a multiplier; thus taking from the equation the numbers belonging to the variable terms, viz. 104, the number of Kreuzers, and $6\frac{5}{6}$, the number of Dollars, we find their product, and multiply it by the fixed number, thus;

$$\frac{1}{3} \ - \ \frac{1}{2} \ - \ 104$$

$$6\frac{5}{6} \quad \text{for 6 D. 25 G.}$$

$$624$$
$$52$$
$$34.66$$

$$\frac{1}{11} \ - \ 710.66$$

$$0.2 \ \frac{1}{11} \ \text{Fixed Number}$$

$$142.132$$
$$6.461$$

$$\text{Batzen} \quad 148.593 \quad \text{Result}$$

We shall now show the application of these principles to the calculation of arbitrated Rates of Exchange between London and Paris, in Bills upon Amsterdam, Hamburg, Frankfort, and Leghorn, and having compared the results in the manner adopted by experienced practical Cambists, we shall further elucidate them by separate statements for each of the most usual operations.

EXAMPLE 1.

From the following rates of Bills in London and Paris, it is required to find,

1st—Whether, having money to transmit from London to Paris, it will be better for me to remit direct Bills to Paris, or for my correspondents in Paris to draw upon me in London, allowing interest for the time the Bills have to run at 4 per Cent per Annum.

2ndly—Whether, having money to draw from Paris, it will be better for my correspondents to make me remittances, or for me to draw upon them.

3rdly—If I have to make remittances to Paris, whether any indirect rate will answer better than the rates of direct Bills.

4thly—If I have to obtain returns from Paris, whether any indirect rate will answer better than either of the direct rates.

RATES OF EXCHANGE.

London, June 17.   Paris, June 14.

25.55	.	.	Paris at 3 months		
12 2	.	.	Amsterdam	. . .	57½
13 10¼	.	.	Hamburg	. . . .	184
151	.	.	Frankfort	. . . .	2 per Cent
48¾	.	.	Leghorn	. . . .	509
			London at 3 months	.	25.10

REDUCTION OF THE DIRECT RATES.

The Direct Paper being at 3 months, the Interest upon it, at 4 per Cent per Annum, is 1 per Cent, or to either rate about 25 Cents.

London on Paris — Fr 25 55 at 3 mo. — Fr 25.30 Short.
Paris on London — Fr 25.10   Do   — Fr 25.35 Short.

WORK OF THE EQUATIONS.

AMSTERDAM.

$$1 \quad \pounds \quad ?$$

1 — * 12   2   Flor. and St. = 12.1 Fl.

Flor. $57\frac{1}{2}$ —   120   Francs.

Fr. 120 × 12.1 ÷ $57\frac{1}{2}$ = 25.25 Francs.

HAMBURG.

$$1 \quad \pounds \quad ?$$

1 — * 13 $10\frac{1}{4}$ Mks and Sc. = $218\frac{1}{4}$ Sc.

Sc. 1600 = Mks 100 — * 184   Francs.

Fr. 184 × $218\frac{1}{4}$ ÷ 1600 = 25.10 Francs.

FRANKFORT.

$$1 \quad \pounds \quad ?$$

1 —   * 151   Batzen W. Z.

90 —   4   Rixdollars.

$103\frac{1}{2}$ —   400   Francs in Frankfort.

4 × 400 ÷ $103\frac{1}{2}$ × 90 = 17176 Fixed Number,

17176   ×   151   = $25.93\frac{1}{2}$

2 per Cent Discount $51\frac{1}{2}$

Francs   25.42 Cents.

LEGHORN.

$$1 \quad \pounds \quad ?$$

1 —   240   Pence.

$48\frac{3}{4}$ * —   1   Pezza.

1 —   * 509   Centimes.

Fr. 5.09 × 240 ÷ $48\frac{3}{4}$ = 25.06 Francs.

COMPARISON OF THE RESULTS OR ARBITRATED RATES

	PRICES.		PARS.	
	London	Paris		
London	25.55	—	25.30	Short
Amsterdam	12 2	57½	25.25	
Hamburg	13.10½	184	25.10	
Frankfort	151	2	25.42	
Leghorn	48¾	509	25.06	
Paris 3 mo	—	25 10	25.35	Short

FOR DIRECT PAPER.

It appears from the direct rates between London and Paris, at 25.30 and 25.35, that

1st. To remit or transfer money from London to Paris, it is better for Paris to draw upon London at 25.35 Short, than for London to remit to Paris at 25.30 Short, because by the former operation there will be made 5 Cents per £, or about ⅕ per Cent more than by the latter.

2ndly. To have returns from Paris, or to remit or transfer money from Paris to London, it is better, by the same 5 Cents, for London to draw upon Paris, than for Paris to remit to London; because the Bills will cost so much less French money, or produce the same proportional part more in Sterling.

FOR INDIRECT PAPER

For remittances to Paris, or to create a fund in Paris, it appears, from the arbitrated results, that Bills upon Frankfort bought in London at 151 Batzen per £ Sterling, and sold in Paris at 2 per Cent Discount, will produce 12 Cents, or very nearly ½ per Cent* more than direct remittances from London to Paris ; or they will produce 7 Cents, which are about

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* If 1-10th per Cent be deducted for extra brokerage, this and the following difference will be reduced to 4-10ths per Cent and 1-16th per Cent.

H

9—32 uds per Cent more than is yielded by direct drafts of Paris upon London.

For returns from Paris, or to create a fund in London, it appears, from the arbitrated results, that Bills on Leghorn bought in Paris at 509 Cents per Pezza, and sold in London at $48\frac{3}{4}$ Pence per Pezza, will cost 29 Cents, or nearly $1\frac{1}{6}$ per Cent less than direct Bills from Paris ; and give a profit of 24 Cents, or nearly 1 per Cent more than will be obtained by Drafts from London on Paris.

In this manner the investigation is conducted, when the Exchanges are tried for a speculation in Bills through intermediate places,* but to determine upon the propriety of making it, independent of the possible changes in the rates, it is necessary to take the charges into consideration.

The regular charges are, 1-10th per Cent for Brokerage upon each purchase or Sale—Stamps upon Drafts—and if the business is conducted by an Agent, one quarter, one third, or one half per Cent according to agreement, for Commission ; but it is not often that business to a great extent of this sort is done but between branches of the same house, or on joint account, the charge for Commission too much diminishing the small rate of profit which such operations commonly even at best afford. The Interest of the money laid out is also to be taken into consideration, but the amount necessarily depends upon the promptness of the return.

To elucidate the preceding calculations and remarks, we will examine the four following suppositions ;

---

* It is to be noticed that it is common in speaking of operations with indirect Paper of any place, to say that the operation is made through that place ; as operations between London and Paris with Amsterdam Paper, are said to be through Amsterdam. However, to entirely coincide with this term, we must suppose that the fund which is to be created in Paris, is formed by our remitting direct Paper to Amsterdam, and either having the proceeds forwarded to Paris, or directing our correspondents in Paris to draw for them upon the Amsterdam House. Either this, or the supposition of the purchase and remittance of Amsterdam Bills to Paris, produces the same arbitrated rate, but the latter operation being less complicated is by far the more common, and it is also attended with fewer charges.

Example 2.

DIRECT REMITTANCES AND DRAFTS.

£ 1000 remitted to Paris, in direct Bills,
at 25.55 will produce . . . . Francs 25550.00
If turned into Cash on arrival, there will be
to deduct 3 months' Interest, which
at 4 per Cent is 255.50

Net—Francs 25294.50

£ 1000 upon London sold in Paris at
25.10 will produce . . . . . Francs 25100.00
Add for the loss of 3 months' Interest be-
fore the bills will be at maturity 251.00

Whole Expense . . 25351.00
(as above) Net Proceeds . . 25294.50

Difference—Francs 56.50

Which difference upon 25000 Francs is about 9-40 ths per Cent.

If, therefore, I wish to create a fund in Paris, I shall obtain more by Paris drawing, than by my remitting.

On the other hand, if I wish to draw money from Paris, it will cost me less to procure £ 1000 by my drawing than by Paris remitting.

The Brokerage is either way the same, and therefore is not brought into the calculation.

EXAMPLE 3.

INDIRECT REMITTANCES.

£ 1000 laid out in London in Bills upon Frankfort at 151 Batzen per £ Sterling, and sold in Paris at 2 per Cent Discount,

will produce at 25.42 . . . . Francs	25490.00
Deduct Brokerage 1 per 1000	25.42
Net	25394.58
Direct Bills, as before, will yield	25294.50
Difference—Francs	100.08

This difference upon 25000 Francs is 1-250 th, or 4-10 ths per Cent profit ; including the charge for extra Brokerage, because direct bills have only one Brokerage, viz. on buying, but indirect bills have one Brokerage on buying and another on selling.

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EXAMPLE 4.

INDIRECT RETURNS.

£ 1000 to be made in London by Bills upon Leghorn bought in Paris at 509 Centimes per Pezza, and sold in London at 48¾ d

per Pezza, will require at 25.06 . . . . . Francs	25060.00
Add for extra Brokerage	25.06
	25085.06
Direct Bills, as before, will require . . .	25351.00
Difference—Francs	265.94

Which profit upon 25000 Francs is about $1\frac{1}{16}$ per Cent.

Example 5.

INDIRECT REMITTANCES AND RETURNS.

£ 1000 laid out in London in Bills on Frankfort, will produce in Paris, as before, in Cash . . . Francs 25394.58

This amount, deducting a further 1 per

Mil. for Brokerage in Paris on buying

returns 25.39

Leaves Net Francs 25369.19

This amount laid out in Paris in the purchase of Bills on Leghorn to be sold in London at the preceding rates, will produce at the arbitrated rate of 25.06 . . £ 1012 6 9

Deduct—Brokerage here on buying and

selling 2 0 3

Interest, 2 Weeks * 1 11 2

Postages, &c. 15 4

Amount of Charges . . 4 6 9

Net £ 1008 0 0

£ 8 Profit on £ 1000 is 4-5 ths per Cent.

These examples form specimens of the most usual operations with arbitrated rates, but they admit of several variations where the Mercantile House has correspondents in different foreign places, by whom the negociations can be conducted.

Commission has not been charged upon these transactions, because they have been considered to be between different branches of the same establishment.—If otherwise, ¼, ⅓, or ½ per Cent, according to agreement, must either be deducted from the proceeds, or applied to the rates according as they are in Sterling or in Foreign money.

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* The supposed average lapse of time between paying for the Bills and receiving the money for the returns.

EXERCISES.

Ex. 1. What are the variations corresponding with 1¼ per Cent in the following rates?

Paris	Frs	25 65 Cts.	Vienna	Flor.	10 5 Kr.
Amsterdam	Flor.	12 3 St.	Venice	Lire	30 30 C.
Hamburg	Mks	13 10 Sc.	Leghorn	Pence	48¼
Berlin	Doll.	7 2 S. G.	Genoa	Lire	25 60 C.
Petersburg	Pence	10⅜	Madrid	Pence	38
Frankfort	Batz.	152	Lisbon	Pence	49¾

2. What rates per Cent do the following differences occasion?

Paris	30	Cents	in Frs	25 55	Cents
Amsterdam	3	Stivers	— Flor.	12 3	Stiv.
Hamburg	4½	Schillings	— Mks	13 10½	Sch.
Berlin	4	Groschen	— Doll.	6 24	Gros.
Petersburg	½	d	— Pence	10⅝	
Frankfort	5	Batzen	— Batz.	152	
Vienna	10	Kreuzers	— Flor.	10 5	Kreuz.
Venice	22½	Centisimi	— Lire	30 20	Cent.
Leghorn	1	d	— Pence	48¼	
Genoa	37½	Centisimi	— Lire	25 20	Cent.
Madrid	½	d	— Pence	38	
Lisbon	½	d	— Pence	49¾	

3. London on Paris at 3 months is 25.65; Paris on London at 3 months is 25.15—If 28 Cents be allowed for interest, what will be the net rates, and the per Centage difference?

4. London on Amsterdam at 3 months is 12 2; Amsterdam on London is 11.95; If the rate of Interest in London is 5 per Cent, and in Amsterdam 2½ per Cent, what will be the net rates, and the per Centage difference?

5. London on Hamburg at 3 months is 13 10½; Hamburg on London at 2 months is 13 8¼; If the rate of Interest in London is 5 per Cent, and in Hamburg 2½ per Cent, what will be the net rates and the per Centage difference?

Ex. 6. Find the arbitrated Rates of Exchange between London and Paris, from the following formulæ of Bills on

AMSTERDAM.

		1	£ ?	
1	—	* 12	1¾	Fl. & S.
57¼*	—		120	Francs.

LEGHORN.

			240	Pence ?
48½	* —		1	Pezza.
100		— *	516	Francs.

HAMBURG.

		1	£ ?	
1	—	* 13	12	Mks & S.
100	—	*	184¼	Francs.

GENOA.

		1	£ ?	
1		—*25.35	Lire N.	
100		— *	100⅙	Francs.

FRANKFORT.

		1	£ ?	
1	—	*	150¾	Batzen.
90	—		4	Rixd.
100	—	* 98		with Dt.
103½	—		400	Francs.

NAPLES.

			240	Pence ?
40⅛	* —		1	Ducat.
100		— *	418	Francs.

MADRID.

			240	Pence ?
37¼	* —		1	Dollar.
4	—		1	Pistole.
1		—*15.50	Francs.	

VIENNA.

		1	£ ?	
1	—	*10	1	Fl. & Kr.
100	—	*	254	Francs.

The rates for direct bills at 3 months being at London 25.60 and Paris 25.12½

It is required to find

1st—For direct Paper, whether it is better for London to remit to Paris, or for Paris to draw on London,

2ndly—For remittances to Paris, and

3rdly—For returns from Paris,

Whether any indirect Paper affords a better rate than direct Paper, and what is the per Centage difference?

Ex. 7. Find the arbitrated Rates of Exchange between London and Amsterdam, from the following formulæ of Bills on

PARIS.

		1 £ ?	
1	—	* 25.60	Francs
120	—	* 57¼	Florins.

LEGHORN.

			240	Pence ?
48½	*	—	1	Pezza.
40		—	* 97¼	Florins.

HAMBURG.

		1 £ ?	
1	—	* 13 12	Mks & S.
40	—	* 35	Florins.

GENOA.

		1 £ ?	
1		— *25.35	Lire.
100		— * 47½	Florins.

FRANKFORT.

		1 £ ?	
1	—	* 150¾	Batzen.
90	—	4	Rixdoll.
20	—	* 36	Florins

NAPLES.

			240	Pence ?
40⅛	*	—	1	Ducat.
40		—	* 79¾	Florins.

MADRID.

			240	Pence ?
37¼	*	—	272	Marav.
375		—	1	Ducat.
40		—	* 100¾	Florins.

VIENNA.

		1 £ ?	
1	—	* 10 1	Fl. & Kr.
30	—	* 36⅛	Florins.

The rates for direct bills at 3 months being, at London 12 1¾ and at Amsterdam 11.90.

It is required to find

1st—For direct Paper, whether it is better for London to remit to Amsterdam, or for Amsterdam to draw on London,

2ndly—For remittances to Amsterdam, and

3rdly—For returns from Amsterdam,

Whether any indirect Paper affords a better rate than direct Paper, and what is the per Centage difference ?

Ex. 8. Find the arbitrated Rates of Exchange between London and Hamburg, from the following formulæ of Bills on

PARIS.

			1	£ ?
.1	—	* 25.60	Francs
187	* —	100	Marks

LEGHORN.

		240	Pence ?
48½*	—	1	Pezza
1	— *	44	Schill.
16	—	1	Mark

AMSTERDAM.

			1	£ ?
1	—	12	1¾	Fl. & St.
35.40	—	40	Marks

GENOA.

		1	£ ?
1	—	*25.35	Lire
185¼*	—	100	Marks

FRANKFORT.

		1	£ ?
1	— *	151	Batzen
90	—	4	Rixdoll.
146½*	—	300	Marks

LISBON.

		240	Pence ?
46¼*	—	1000	Reis
1000	— *	42¼	Schill.
16	—	1	Mark

VIENNA.

			1	£ ?
1	—	10	1	Fl.&Kr.
146¼*	—	200	Marks

MADRID.

		240	Pence ?
37½*	—	272	Marav.
375	—	* 46⅞	Schill.
16	—	1	Mark.

The rates for direct Bills at 3 months being at London 13 12 and at Hamburg 13 8¼

It is required to find

1st, For direct Paper, whether it is better for London to remit, or Hamburg to draw.

2ndly, For remittances to Hamburg, and

3rdly, For returns from Hamburg,

Whether any indirect Paper affords a better rate than direct Paper, and what is the per Centage difference.

Ex. 9. If I remit £ 1000 in Paper to Paris at 25.55 at 3 months, and it is discounted at 5 per Cent for 86 days, what net credit shall I receive ?

10. If I buy a Bill in London on Rotterdam for Florins 5000 at 12 1½, and sending it to Paris it is negociated there at 57¼, what net credit shall I receive ; allowing Brokerage 1-10 th per Cent, and Commission 1-3 rd per Cent; and what rate of Exchange will be established between London and Paris, allowing a Brokerage upon the Purchase here of 1-10 th per Cent?

11. If I buy a Bill for Francs 10.000 in London at 25.50, and send it to Amsterdam and get it negociated there at 57½, what net credit shall I receive, and what rate of Exchange will be established, supposing the rate of the charges to be the same as before ?

12. If I invest £ 1000 in Bills upon Hamburg at 13 12¼, and I send them to Amsterdam to be sold there, what will be the net proceeds, supposing the selling price to be 35⅜, and the whole charges on the sale to be ½ per Cent? And what profit or loss per Cent shall I make by this operation, if I have Bills in return at 2 months' date at 11.90 ; deducting charges on the purchase at Amsterdam ½ per Cent, and allowing for Brokerage in London 1 per 1000, besides Interest for 67 days at 5 per Cent per Annum ?

<hr>

PRODUCTS.

Ex. 1.	Paris	32.06	Cts	Vienna	7.56¼	Kr.
	Amsterdam	15.18¾	Cts	Venice	37.87½	Cent.
	Hamburg	2.72½	Sc.	Leghorn	2.41¼	4 ths
	Berlin	2.65	Gr.	Genoa	32.—	Cent.
	Petersburg	1.01¼	8 ths	Madrid	1.9	4 ths
	Frankfort	1.9	Batz.	Lisbon	2.48¾	4 ths

PRODUCTS CONTINUED.

Ex. 2. Paris	1.17 per Cent.	Vienna	1.65 per Cent.
Amsterdam	1.23 —	Venice	0.74 —
Hamburg	2.05 —	Leghorn	2.07 —
Berlin	1.96 —	Genoa	1.48 —
Petersburg	4.70 —	Madrid	1.31 —
Frankfort	3.28 —	Lisbon	1.00 —

3. London Net 25.37   Paris   Net 25.43 —Diff. $\frac{1}{4}$ per Ct.

4. London — 11 19   Amsterdam — 12.02$\frac{1}{2}$ — $\frac{5}{8}$ —

5. London — 13 7$\frac{3}{4}$ Hamburg — 13 9$\frac{1}{8}$ — $\frac{11}{18}$ —

6. Net direct rates   London   25 34.4 — Paris 25 37.6

Indirect rates

Amsterdam	25 33.6 Cts.
Hamburg	25 33.4
Frankfort	25 37.5
Vienna	25 44.2
Leghorn	25 53.4
Genoa	25 38.1
Naples	25 00.1
Madrid	24 96.6

For direct Bills, Paris should draw — Difference, 1-8 th per Ct.

— Indirect remittances, Leghorn is 19 Cts. or $\frac{3}{4}$ per Ct. better.

— Indirect returns, Madrid is 37.8 — 1$\frac{1}{2}$ per Ct. better.

7. Net direct rates London   11 96.6 Amsterdam 12 01.9

Indirect rates

Paris	12 21.3
Hamburg	12 03.1
Frankfort	12 06.0
Vienna	12 06.1
Leghorn	12 03.1
Genoa	12 04.1
Naples	11 92.5
Madrid	11 77.0

Ex. 7. Continued.—For direct Bills, Amsterdam should draw.
Difference 7-16ths per Cent.
—Indirect remittances, Paris is 24,7 Cts. or 2 per Ct. better.
—Indirect returns, Madrid is 24,9 ―― 2 per Ct. better.

8. Net direct rates London   13   9.8 Hamburg 13 10.4

Indirect rates	Paris	13	11.0
	Amsterdam	13	10.5
	Frankfort	13	11.8
	Vienna	13	11.1
	Leghorn	13	9.7
	Genoa	13	10.9
	Lisbon	13	11.2
	Madrid	13	11.0

For direct Bills, Hamburg should draw. Diff. 3-11ths per Ct.
—Indirect remitt. Frankfort is 2 Sch. or $\frac{10}{11}$ per Ct. better.
—Indirect returns Leghorn is $\frac{7}{10}$ Sch. or $\frac{7}{22}$ per Ct. better.

9 Net Credit—Francs   25244 82 Cents. *

10. Cost in London £ 414   9 10—Net Proceeds in Paris.
Francs   10434.94 — Rate of Exchange. Francs 25.17$\frac{1}{2}$

11. Cost in London £ 392 11   0 Net Proceeds in Amsterdam.
Florins 4770.91 — Rate of Exch. Flor. 12 15 Cts. or 3 Stiv.

12.			
Amount of the Bill	Bco Mks	13765 10	Sch.
Proceeds in Amsterdam	Florins	12173 97	Cts.
Net amount of returns	Florins	12052 54	Cts.
Net amount in Sterling	£	1012 16	4
Interest and Brokerage		10 6	0

Profit—£ 2 10 4 or $\frac{1}{4}$ per Cent.

---

* N. B. 360 days are generally reckoned 1 Year, in calculations of this nature.

COMPOUND ARBITRATIONS.

The rate of exchange between two places, produced by remittances through an intermediate place, in any other than direct Paper of that place, is called a compound arbitrated rate.

Thus, as has before been stated, if Bills upon Amsterdam are sent from London to Amsterdam, and the proceeds are sent to Paris in direct Bills, the rate thus effected between London and Paris is called a simple arbitrated rate ; but in such remittances through Amsterdam, if Hamburg Bills are sent either from London to Amsterdam, or from Amsterdam to Paris, the rate thus produced between London and Paris, is called a compound arbitrated rate.

Operations of this nature are also called circuitous, when either more than three places are concerned, or, more properly, when the proceeds of the Bills, after having passed through two or more other places, return to the original place ; but the houses capable of thus extending their negociations, are so few, and the liability to unfavourable changes becomes so much increased, that in the practice of exchanges they are of very limited occurrence.

The calculations of compound rates are, like those of simple arbitrations, usually performed by the Chain Rule ; for remittances, the first of the principal rates is the buying price at the place where the operation commences; the next, the selling price at the intermediate place if direct Bills are not used ; the next, the buying price of either direct or indirect Bills ; and, lastly, the selling price at the place where the operation terminates if it receives indirect Bills.

EXAMPLE 1.

LONDON AND PARIS.

Direct Bills from London   Direct from Amsterdam.

If I buy a bill upon Amsterdam at 12 Flor. 2 St. per £ Sterling and send it to Amsterdam, and the proceeds are forwarded to Paris at 57½ Florins per 120 Francs, what proportional exchange does this remittance establish between London and Paris ?

			1	£ ?	
	1	—	⁂ 12 10	Flor. and Cents.
Flor. 57½	⁂	—	120	Francs.
			12.10		
	57½		1452.00		
	115	)	2904.00 (25.25		
			604		
			290		
			600		

Result—Francs   25 25 Cents.

Upon referring to page 96, we find this to be the same calculation that is there made for the purchase of Netherland Bills in London, and the sale of them in Paris ; this is therefore a simple arbitration, and it is introduced here only because it shows the same routine of the operation with direct Bills, as the three following examples with indirect Bills, which the calculation referred to does not.

Instead of forwarding the proceeds from Amsterdam to Paris, the same result, independent of charges, will come out, by Paris drawing upon Amsterdam on the credit of the fund created there.

EXAMPLE 2.

LONDON AND PARIS.

Indirect from London   Direct from Amsterdam.

If I buy a bill upon Hamburg at Mks 13 10½ Sc. per £ Sterling, and I send it to Amsterdam, and it is sold there at 35½ Florins per 40 Banco Marks ; and if the proceeds are remitted to Paris in French Bills at 57½ Florins per 120 Francs, what price does this transaction establish between London and Paris ?

```
                                    1   £   ?
                  £  1    —    * 13 10½ = 218½ Sch.
    Sch. 640 = Mks 40     —       * 35½  Florins.
                  57½ *   —        120   Francs.
                                   35½
                                  ————
                 640              4260
                 57½             218½
                ————            —————
                4480             76680
                3200             8520
                 320             2130
                ————            —————
                368.00   )       9308.10 ( 25.29
                                 1948
                                 1081
                                 3450
```

Result—Francs   25 29 Cents.

The first rate, here, is the buying price in London, the second is the selling price at Amsterdam, and the third and last is the buying price of French Bills at the same place, the remittance thence being made to Paris in direct Bills.

N. B. This calculation may be made as in Ex. 4, page 113, without reducing the Marks into Schillings.

EXAMPLE 3.

LONDON AND PARIS.

Direct from London   Indirect from Amsterdam.

If I buy a Bill upon Amsterdam at 12 2, and it is forwarded to Amsterdam, and the proceeds are laid out in Bills upon Hamburgh at 35½, which are forwarded to Paris, and sold there at 185 Francs per 100 Marks Banco, what rate does this operation establish between London and Paris ?

			1	£ ?
	1	—	* 12 10	Flor. and Cts.
	35½ *	—	40	Bco Mks
	100	—	* 1.85	Francs.
			40	
			74.00	
			12.1	
			7400	
			888	
	35½		895.400	
	2		2	
	71	)	1790.800 (25.22	
			370	
			158	
			160	

Result—Francs 25   22 Cents.

The buying price in London is the first rate, the buying price at Amsterdam the second rate, and the selling price at Paris the third rate.

EXAMPLE 4.

LONDON AND PARIS.

Indirect from London   Indirect from Amsterdam

If I buy a Bill upon Hamburg at 13 10½, and it is sold in Amsterdam at 35; if also the proceeds are there laid out in Bills upon Genoa at 46½ Florins per 100 Lire, and these Bills are sold in Paris at 1 per Cent Discount, what rate does this operation establish between London and Paris?

			1	£	?
1	—	* 13	10½	Mks and Sch.
40	—		* 35	Florins.
46½ *	—		100	Lire.
100	—		* 99	Francs.

	3500	
	13	
	45500	
	1750	for 8
46½	437.5	— 2
40	109.37	— ½
1860)	47796.87 (25.69	
	1059	
	1296	
	808	

	Fr.	C.	
	25	69	
Discount		25	1 per Ct.
Result—Francs	25	44	Cents.

Example 5.

CIRCUITOUS EXCHANGE.

LONDON, PARIS, AND LONDON.

Suppose I buy Bills upon Hamburg at 13 10½, and they are sold in Amsterdam at 35; that the proceeds are invested in Bills upon Genoa at 46½, and that these Bills are transmitted to Paris and sold at 1 per Cent Discount; and lastly, that the proceeds are laid out in the purchase of Bills upon Madrid at 15 Frs 20 Cts per Pistole of Plate, and that these Bills are transmitted to me and sold at 36¾ d per Dollar.

What Profit or Loss per Cent would this circuitous exchange produce, independent of the charges?

			100	£ ?
1	—	13	10½	= 218½ Sch.
Sch. 640 = Mks 40	—		35	Florins.
46½	—		100	Lire.
100	—		99	Francs.
15.20	—		1	Pistole = 4 Doll.
1	—		36¾	Pence.
240	—		1	£ Sterling.

$$\frac{218\frac{1}{2} \times 35 \times 99 \times 4 \times 36\frac{3}{4}}{64 \times 46\frac{1}{2} \times 15.20 \times 24} = £\ 102.514$$

Result   £   102   10   3

Profit   £   2   10   3 per Ct.

To form a proper estimate of the Profit or Loss upon these exchange transactions, the charges must be applied thus:

To the first rate	Fr	25.25	
Double Brokerage — 2 per 1000 . . Cts	5		
One Commission — say ¼ per Cent . . .	6	. 11	
Net	Fr	25.14	

To the second rate	Fr	25.29	
3 Brokerages — 3 per 1000 . . . . . Cts	7½		
Double Commission — ½ per Cent . . .	12½	. 20	
Net	Fr	25.09	

To the third rate	Fr	25.22
3 Brokerages and double Commission		20
Net	Fr	25.02

To the fourth rate	Fr	25.44	
4 Brokerages — 4 per 1000 . . . Cts	10		
4 Commissions — 1 per Cent	25	. 35	
Net	Fr	25.09	

The last calculation for finding the Profit per Cent, requires correction in the following manner:

Arbitrated Profit per Cent.		£ 2	10	3
6 Brokerages — 2 at each place . £ 0 12 0				
4 Commissions at ¼ per Cent each . . 1 0 0				
Interest for 3 weeks, postages, &c. . . . 7 3	1	10	3	
Net Profit per Cent	£ 0	11	0	

In each arbitration, there is one brokerage to be deducted on each purchase, and one on each sale.—There is also a Commission, here reckoned at ¼ per Cent, upon each sale and upon each purchase, at Amsterdam and Paris.

If the houses abroad are connected with the London House, the commissions will be saved, or rather divided, and the speculation will be so much the more advantageous: but it is seldom that any great per Centage profit can be made by such transactions, unless by unexpected improvements in the rates of exchange.

REMARKS ON THE APPLICATION OF PER CENTAGE CHARGES.

In the calculation of the charges on exchange operations distinctions are sometimes made, as to whether the per Centages are to be reckoned upon the given amount, or upon that which is produced by the addition or subtraction of the per Centage amounts.

Thus supposing the charges to be 3 per Cent, and the given amount £ 1000, a distinction is made as to whether the proportional statement should be in this form.

If £ 100 produce £ $\left\{ \begin{array}{c} 103 \\ \text{or} \\ 97 \end{array} \right\}$ what will £ 1000 produce ?

To which the product will be £ 1030 or £ 970; or in this form.

If £ $\left\{ \begin{array}{c} 97 \\ \text{or} \\ 103 \end{array} \right\}$ produce £ 100 what will £ 1000 produce ?

To which the product will be nearly £ 1030 18 7, or £ 970 17 6 ; the former two products making the differences, each £ 30, to be 3 per Cent on the given amount £ 1000, and the latter two products making the differences, £ 30 18 7 and £ 29 2 6, to be 3 per Cent on their amounts £ 1030 18 7 and £ 970 17 6.

But in the finding of an arbitrated Rate of Exchange, it is but very seldom that these distinctions are necessary, because the amounts both of the Rates and Charges are too small to make the difference of any consequence. When otherwise, instead of working the Proportion, a correction may be made upon the first per Centage amount, by adding or subtracting the same per Centage upon that amount.

Thus to apply so much even as 3 per Cent to one of the largest rates with London, as the rate with France at 25.60

$$
\begin{array}{lr}
\text{3 per Cent upon 25.60 is} & . \quad . \quad 76.8 \text{ Cents.} \\
\text{correction—3 per Cent upon 76 Cents is} & . \quad 2.2 \\
\hline
\text{making by addition} \quad . \quad . \quad . & 79.0 \\
\hline
\text{or, \quad by subtraction} \quad . \quad . \quad . & 74.6 \\
\hline
\end{array}
$$

If therefore 3 per Cent is to be calculated upon the total, as in the 1 st form 97 to 100, we shall obtain the result thus :

$$
\begin{array}{lr}
\text{Net rate} \quad . \quad . & 25.60 \\
\text{Add} \quad . \quad . \quad . \quad . & .79 \\
\hline
\text{Full rate} \quad . \quad . & 26.39 \\
\hline
\end{array}
$$

Which is the same as is produced by working the proportion ; for 97 is to 100, as 25.60 to 26.39 ; or in other terms 3 per Cent on 26.39, which is to be found, is 79 Cents.

On the other hand, 3 per Cent in the 2 nd form 103 to 100, is found by subtraction : thus,

$$
\begin{array}{lr}
\text{Full rate} \quad . \quad . & 25.60 \\
\text{Subtract} \quad . \quad . & 74\tfrac{6}{10} \\
\hline
\text{Net rate} \quad . \quad . & 24.85\tfrac{4}{10} \\
\hline
\end{array}
$$

Which is the same as is produced by working the proportion, for 103 is to 100 as 25.60 to 24.85$\tfrac{4}{10}$; or 3 per Cent on 24.85 is very nearly 74$\tfrac{6}{10}$ Cents.

If the rate of the per Centage is less, the correction becomes of less consequence, and may usually be disregarded.

It must further be observed, that in exchange circulations the charges are, in practice, calculated at each step ; but this precision in finding an arbitrated rate, is wholly unnecessary.

We shall conclude this subject by showing the routine of the last supposition, Example 5.

ROUTINE OF THE CIRCUITOUS ARBITRATED RATE.

£ 1000 laid out in London in Bills on Hamburg
at 13 10½ produce . . Banco Mks 13656 4 Sc.

These Bills transmitted to Amsterdam and sold
at 35 Flor. per 40 Marks produce - - Florins 11949.22
deduct, Brokerage for selling Hamburg
Bills and buying Genoa Bills - Flor. 23.89
Two Commissions each ¼ per Cent - 59.75
Postages, &c. - - - - - - 3.58 87.22

Net proceeds - Florins 11862. 0

This amount laid out in Bills on Genoa
at 46½ Flor. per 100 Lire produces Lire 25509 67 Cent.

These Bills sold in Paris at
1 per Cent discount produce - - - Francs 25254.58
deduct, Brokerage for selling Bills on Genoa,
and buying Bills on Madrid - - Frs 50.50
Two Commissions each ¼ per Cent - 126.27
Postages, &c. - - - - - - 7.23 184.00

Net proceeds - Francs 25070.58

This amount laid out in Bills on Madrid,
at Fr. 15.20 per Pistole produces D. P. 6597 4 6

These Bills sold in London at 36¾ d
per Dollar, produce in Sterling - - - £ 1010 4 11
deduct Brokerage for buying Hamburg
Bills and selling Spanish Bills - £ 2 0 0
Interest, 3 weeks on £ 1000 at 4 per Ct. 2 6 2
Postages, &c - - - - - - 8 9 4 14 11

Net profit - £ 5 10 0

£ 5 10 s upon £ 1000 are 11-20 ths per Cent.
The estimate in page 115 gives the same result.

EXERCISES.

What Proportional Exchange is established between,

Ex. 1. London and Paris by Bills upon Amsterdam, bought in London at 12 1½; the proceeds having been forwarded from Amsterdam to Paris, in direct Bills at 56½ Florins per 120 Francs ?

2. London and Paris by Bills upon Hamburg, bought in London at 13 11¾, and sold in Amsterdam at 35½ Florins per 40 Marks, the proceeds having been forwarded to Paris, in direct Bills at 56½ Florins per 120 Francs ?

3. London and Paris by Bills upon Amsterdam, bought in London at 12 1½ ; the amount having been forwarded from Amsterdam to Paris in Bills upon Hamburg, which Bills were bought in Amsterdam at 35½ Flor. per 40 Banco Marks, and sold in Paris at 186½ Francs per 100 Marks ?

4. London and Paris by Bills upon Hamburg, bought in London at 13 11¾, and sold in Amsterdam at 35½ Florins per 40 Marks Banco; the proceeds having been invested in Bills upon Genoa at the rate of 46¾ Flor. per 100 Lire, which Bills were sold in Paris at ⅝ per Cent Premium ?

5. London and Hamburg by Bills upon Frankfort, bought in London at 151¾, and remitted to Paris and sold at 1¼ per Cent Discount; the proceeds having been forwarded to Hamburg, in direct Bills at 186½ Francs per 100 Marks Banco ?

6. London and Amsterdam by Bills upon Vienna, bought in London at 10 2, and remitted to Paris and sold there at 253½ Francs per 100 Florins; the proceeds having been remitted to Amsterdam in Bills upon Frankfort, which were bought in Paris at 1¼ per Cent Discount, these latter Bills having been sold in Amsterdam at 36¼ Florins per 20 Rixdollars W. Z. ?

Ex. 7. London and Frankfort by Bills upon Hamburg, which were bought in London at 13 11¾, and having been remitted to Amsterdam were sold there at 35¼ Florins per 40 Marks Banco; the proceeds having been forwarded to Frankfort in Bills upon Augsburg, which were bought in Amsterdam at 36 Netherland Florins per 30 Florins of Augsburg, and sold in Frankfort at 100½ Rixdollars W. Z. per 150 Florins of Augsburg.—The charges being at Amsterdam, ½ and 2-10 ths per Cent for Commission and Brokerage, and for the same at Frankfort 1-3 rd and 1-10 th per Cent; besides interest from the 28th of September to the 12th of October at 5 per Cent per Annum ?

8. London and Frankfort by Bills upon Paris, which were bought in London at 25.65 and sold in Amsterdam at 56½; the proceeds having been invested in Bills upon Genoa bought at 46¾, and sold in Hamburg at 186; and the proceeds having thence been transmitted to Frankfort in Bills upon Berlin bought at 152½, and sold in Frankfort at 104½—The charges being 3 Commissions at 1-3 rd per Cent each, and 5 Brokerages at 1-10 th per Cent each, besides Interest for 20 days at 4 per Cent per Annum ?

PRODUCTS.

Ex. 1. Fr. 25.64$\frac{6}{10}$ Ex. 2. Fr. 25 88$\frac{8}{10}$ Ex. 3. Fr. 25.37$\frac{4}{10}$
4. Fr. 26.23$\frac{5}{10}$ 5. Mks 13 12$\frac{8}{10}$ Sc.. 6. Fl. 12.07$\frac{9}{10}$

7. Rate without deduction . . Batzen 153.127
   Allowance for charges at Amsterdam —   1.071
      Do - Do - at Frankfort —   .658
  Interest . . . . . . —   .204
         Net Rate — 151.104

8. Rate without deduction . . Batzen 154.262
  Allowance for Commiss. and Brok. —   2.313
  Interest . . . . . . —   .342
        Net Rate — 151.607

BANKING OPERATIONS,

OR

THE COMPARISONS OF RATES OF EXCHANGE

ON

DIFFERENT PLACES.

In the execution of orders for Remittances or Drafts, it is common for limited prices of Bills on different places to be given to the correspondent, to guide him in the choice of the Paper which he should purchase or sell for his principal ; and if, as is very common, the limits of the orders and the present prices do not agree, it becomes necessary to compare the given rates with the present rates, in order to determine whether the order should be executed, or amongst several rates, which should be preferred.

The principles upon which these comparisons are founded, are the same as those before explained ; viz. that

For Remittances—That rate is the best of which the variable price is the highest in Foreign money, or the lowest in the money of the place making the operation ; and that

For Drafts—That rate is the best of which the variable price is lowest in Foreign money, or the highest in the money of the place making the operation.

The comparisons being here however made between rates on different places, and not between arbitrated rates upon the same place, different formulæ are required to be employed, the application of which is facilitated by use of the following letters or symbols.

Fixing the place of operation at London, the money of which is Sterling, we use

S to signify a Sterling Rate, as that on Madrid.

F . . . . Foreign Rate, ... Paris.

g . . . . . . the given price, or limit of the order.

p . . . . . . the present price, or that at which the order can be executed.

Then for the purpose of affording the means of comparing rates in different monies, and with different fixed prices, it becomes necessary to express them fractionally; that is, to compare them as abstract numbers with unity; in the performance of which we are directed, by the well known arithmetical rule, that, the other term of a fraction remaining fixed, the fraction increases as either the numerator increases, or the denominator decreases; and that the fraction decreases when the reverse takes place.

Applying these symbols to the preceding principles, upon which the comparisons are stated to be founded, viz, that

For Remittances with Foreign rates (F), the greater p becomes with respect to g, or, with Sterling rates (S), the less p becomes with respect to g, the more the rate improves—and, on the other hand,

For Drafts with Foreign rates (F), the less p becomes in comparison with g, or, with Sterling rates (S), the greater p becomes in comparison with g, the more the rate improves.

And employing also the fractional forms, we have these formulæ:

FOR REMITTANCES

With $\left\{\begin{array}{c} F \\ S \end{array}\right\}$ Rates, the rate improves as $\left\{\begin{array}{c} \dfrac{p}{g} \\ \dfrac{g}{p} \end{array}\right\}$ increases.

FOR DRAFTS.

With $\left\{\begin{array}{c} F \\ S \end{array}\right\}$ Rates, the rate improves as $\left\{\begin{array}{c} \dfrac{g}{p} \\ \dfrac{p}{g} \end{array}\right\}$ increases.

For comparing the results of these formulæ, it is generally better to reduce the fractions into decimals, as the greatest result can then be more easily perceived; and in comparing the fractions for Remittances with those for Drafts, to find whether a gain in the one compensates for a loss in the other, it is

generally sufficient to find whether the excess above unity of the one, is equal or superior to the deficiency below unity in the other, though perfect accuracy would require the product of the fractions to be taken, and compared with unity.

We shall now show the application of these formulæ to the most usual operations; dividing them into two portions; viz. 1 st, the comparison of the given and present rates for Remittances and Drafts, both separately and together, with Sterling and Foreign rates; and, 2 ndly, the finding of an equivalent present rate, when only one of the present rates is given.

N. B. In the practice of these comparisons, it is usual to employ the terms, Cash and Bills, instead of Remittances and Drafts; thus, as the Rate on Paris increases, it is said to be better for Cash, or the laying out of money, and worse for Bills, or for drawing : while, on the reverse, as this Rate decreases, it is worse for Cash and better for Bills.

On the Continent it is usual to character the Courses of Exchange with the same or similar terms; as, *Argent, Papier, Geld, Briefe;* or *Denaro, Lettera,* &c. the Cash rates being those at which Bills were bought of the Bankers, and the Bill rates those at which they were sold by the same parties ; and the same terms are also employed to denote the state of the Bill Market, when the rates are expressed in only one or the other Column; Cash then signifies that Bills are in demand, and Paper that the supply is greater than the demand.

EXAMPLE 1.

ORDER FOR REMITTANCES.

I have an order to execute from Frankfort, for Bills either upon Hamburg at 13 10, upon Amsterdam at 12 1, upon Paris at 25.60, or upon Leghorn at 48, or in the Paper that best agrees with these prices.—On the present exchange-day the prices of these Bills are, Hamburg 13 $9\frac{1}{2}$—Amsterdam 12 0—Paris 25.50—and Leghorn $48\frac{1}{2}$.—Which Paper is the best for me to remit?

$$\text{F. Hamburg.} \quad \frac{p}{g} \quad \frac{13\frac{12}{32}}{13\frac{9}{16}} = \frac{435}{436} = 0.997\frac{1}{2}$$

$$\text{F. Amsterdam.} \quad \frac{p}{g} \quad \frac{12}{12\frac{1}{20}} = \frac{240}{241} = 0.995$$

$$\text{F. Paris.} \quad \frac{p}{g} \quad \frac{25.50}{25.60} = 0.996$$

$$\text{S. Leghorn.} \quad \frac{g}{p} \quad \frac{48}{48\frac{1}{2}} = \frac{96}{97} = 0.989$$

The best of these Papers is that upon Hamburg, which is only $2\frac{1}{2}$ per 1000, or $\frac{1}{4}$ per Ct. less than the limited given price. Paris Paper is the next in value, and the worst is that upon Leghorn.

When the differences in the respective values are not so minute, as to require the trouble of valuing the fractions in order to find out which is the best, they may be calculated thus;

Hamburg	13 10	and	13 $9\frac{1}{2}$	diff.	$\frac{1}{2}$ Sch.	or	$\frac{1}{4}$ per Ct.
Amsterdam	12 1	—	12 0	—	1 Stiv.	—	42-100 ths
Paris	25.60	—	25.50	—	10 Cts	—	39-100 ths
Leghorn	48	—	$48\frac{1}{2}$	—	$\frac{1}{2}$ d	—	1 per Ct.

EXAMPLE 2.

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ORDER FOR DRAFTS.

I have an order from Frankfort to draw upon either of the following places, at the following rates, or the best that the changes in the course may allow; viz. Hamburg at 13 12, Amsterdam at 12 0, Paris at 25.40, or Leghorn at $48\frac{3}{4}$, and the prices of Bills in London, now, are on Hamburg 13 $13\frac{1}{4}$, Amsterdam 12 2, Paris 25.50, and Leghorn $48\frac{1}{2}$. Upon which place should I draw in conformity with my instructions?

$$\text{F. Hamburg.} \quad \frac{g}{p} \quad \frac{13\frac{12}{16}}{13\frac{53}{64}} = \frac{880}{885} = 0.994$$

$$\text{F. Amsterdam.} \quad \frac{g}{p} \quad \frac{12}{12\frac{1}{10}} = \frac{120}{121} = 0.991$$

$$\text{F. Paris.} \quad \frac{g}{p} \quad \frac{25.40}{25.50} = 0.996$$

$$\text{S. Leghorn.} \quad \frac{p}{g} \quad \frac{48\frac{1}{2}}{48\frac{3}{4}} = \frac{194}{195} = 0.994$$

Answer, on Paris.

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The Foreign rates having risen, and the Sterling rate having fallen, all have become worse for drawing, but that on Paris the least.

N. B. It is evident in this fractional arrangement of the prices, in both this and the last operation, that if either result is greater than unity, that rate has improved beyond the given limit for either purpose, and that if more than one rate has improved, that rate is to be taken which has improved the most.

EXAMPLE 3.

ORDER FOR REMITTANCÈS AND DRAFTS.

I have an order from Hamburg to remit Bills on Amsterdam at 12 2, and to draw upon Paris at 25.45, or at equivalent rates.—The present rates are 12 2½ on Amsterdam, and 25.50 on Paris, should I execute the order ?

$$\text{F. Hamburg} \quad \text{Remit} \quad \frac{p}{g} \quad \frac{12\frac{5}{10}}{12\frac{1}{10}} = \frac{485}{484} = 1.002$$

$$\text{F. Paris} \quad \text{Draw} \quad \frac{g}{p} \quad \frac{25.45}{25.50} = \frac{509}{510} = 0.998$$

It here appears that the betterness of the fraction for the Remittance, is equal to the worseness of that for the Draft, and therefore the order may be executed.

If more exactness is requisite, the decimals may be extended, as to 1.00206 and 0.99804* and they then show, that the alterations taken together make the transaction more favourable than the limits of the order.

But the same may be perceived by the fractions alone, for the difference of the first from unity is 1-484 th, and that of the second 1-510 th, and therefore as the former is greater than the latter, the Remittance and Draft may be executed with advantage.

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* It has been remarked, that precision may require the fractions or their values to be multiplied together, thus

$$1.00206 \times 0.99804 = 1.000095 \&c.$$

but in this example the product is very nearly the same as the sum of the two fractions.

EXAMPLE 4.

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ORDER FOR REMITTANCES AND DRAFTS.

I am ordered by my correspondent at Paris to remit him Paper on Hamburg at 13 11½, and to reimburse myself by drawing on Leghorn at 48¼.—The rates here being on Hamburg 13 11, and on Leghorn 48½, should I execute the order?

Mks 13 11½ = 219½ Sch. Mks 13 11 = 219 Sch.

F. Hamburg Remit $\dfrac{p}{g}$ $\dfrac{219}{219\frac{1}{2}}$ = 0.997¾

S. Leghorn Draw $\dfrac{p}{g}$ $\dfrac{48\frac{1}{2}}{48\frac{1}{4}}$ = 1.005

The rate for Drafts appears to have improved more than the rate for Remittances has deteriorated, and it will therefore be proper to execute the order.

The same may be found from per-centaging the differences in the rates; thus, the Hamburg difference, ½ Sch. Banco, is 2-9 ths per Cent Worse; and the Leghorn difference, ¼ d Sterling, is ½ per Cent Better, and consequently the improvement is greater than the deterioration.

It is evident, in any case, that if the improvement is less than the deterioration, the order should not be executed.

TO FIND EQUIVALENT RATES.

When there are limitations in the prices of compound orders, or those for drawing or remitting, it is frequently necessary, from comparing one of the present rates with its correspondent given rate, to find the other equivalent rate, which may be done, by the following directions :

Arrange the fractional form of the two sets of prices according, as before, to whether the rate at London is in Foreign money or Sterling, leaving that term blank * which is required ; then,

If a numerator is wanting, multiply the denominators together, and divide the product by the single numerator ; or

If a denominator is wanting, multiply the numerators together, and divide the product by the single denominator.

EXAMPLE 1.

I have orders to remit upon Paris at 25.60, and to draw upon Amsterdam at 12 2. The rate upon Paris has fallen to 25.50, at what equivalent price must be the rate upon Amsterdam, to prevent any loss ?

The Rates for both these places are Foreign.

$$
\text{F. Paris} \quad \text{Remit} \quad \dfrac{p}{g} \quad \dfrac{25.50}{25.60}
$$

$$
\text{F. Amsterdam} \quad \text{Draw} \quad \dfrac{g}{p} \quad \dfrac{12.10}{x}
$$

$$
\frac{12.10 \times 25.50}{25.60} = 12.05 \text{ Answer.}
$$

Hence, as the course by falling has become unfavourable for remitting, the course upon Amsterdam must be improved, also by falling to a proportionate rate ; viz. Flor. 12 1 St.

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* In the following formulæ, the blank term is denoted by the letter x, and it may be here observed, that when this letter occurs in statements of Equations, Proportions, &c. it must be understood as the Algebraical expression for the required quantity.

EXAMPLE 2.

I have orders to remit upon Paris at 25.60, and to draw upon Amsterdam at 12 2.—If the Amsterdam rate improves for drawing to 12 1, to what depreciation in the Paris rate shall I be limited?

Rates as Ex. 1.

			p	x
F.	Paris	Remit	$\dfrac{p}{g}$	$\dfrac{x}{25.60}$
			g	12.10
F.	Amsterdam	Draw	$\dfrac{g}{p}$	$\dfrac{12.10}{12.05}$

$$25.60 \times 12.05 \div 12.1 = 25.49 \text{ Answer.}$$

The remittance may therefore be executed as low as 25.50, without making the given limits worse.

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EXAMPLE 3.

I have to remit upon Madrid at 38, and to draw upon Leghorn at 48.—The Madrid rate having risen to $38\frac{1}{4}$, to what rate shall I be limited for Leghorn?

The rates for both these places are Sterling.

			g	38
S.	Madrid	Remit	$\dfrac{g}{p}$	$\dfrac{38}{38\frac{1}{4}}$
			p	x
S.	Leghorn	Draw	$\dfrac{p}{g}$	$\dfrac{x}{48}$

$$38\frac{1}{4} \times 48 \div 38 = 48\frac{5}{16} \text{ Answer.}$$

Hence, from the Madrid rate having got worse for remitting, I must not execute the order unless the Leghorn rate rises above $48\frac{5}{16}$ d, as otherwise I shall not obtain enough for the Bills on Leghorn, to answer the proportionate advance of the Bills on Madrid.

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EXAMPLE 4.

I have to remit on Madrid at 38, and draw upon Leghorn at 48—If the Leghorn rate improves for drawing to 48¼, to what extent may I go with Madrid?

Both rates are Sterling.

S.	Madrid	Remit	$\dfrac{g}{p}$	$\dfrac{38}{x}$
S.	Leghorn	Draw	$\dfrac{p}{g}$	$\dfrac{48\frac{1}{4}}{48}$

$$38 \times 48\tfrac{1}{4} \div 48 = 38\tfrac{3}{16} \text{ Answer.}$$

The Leghorn rate having improved, I shall get more for my Drafts, and can therefore afford to give more for the Bills upon Madrid should this rate also have risen.

EXAMPLE 5.

I have to remit upon Leghorn at 48, and draw upon Paris at 25.60—If the Paris rate improves to 25.50, to what rate shall I be limited upon Leghorn?

London upon Leghorn   —   Sterling rate.
London upon Paris   —   Foreign rate.

S.	Leghorn	Remit	$\dfrac{g}{p}$	$\dfrac{48}{x}$
F.	Paris	Draw	$\dfrac{g}{p}$	$\dfrac{25.60}{25.50}$

$$25.60 \times 48 \div 25.50 = 48\tfrac{3}{16} \text{ Answer.}$$

EXAMPLE 6.

I have to remit upon Leghorn at 48, and to draw upon Paris at 25.60—If the Leghorn rises to $48\frac{1}{4}$, at what rate may I draw upon Paris ?

S.	Leghorn	Remit	$\dfrac{g}{p}$	$\dfrac{48}{48\frac{1}{4}}$
F.	Paris	Draw	$\dfrac{g}{p}$	$\dfrac{25.60}{x}$

$$25.60 \times 48 \div 48\frac{1}{4} = 25.46 \text{ Answer.}$$

EXAMPLE 7.

If I have to remit upon Paris at 25.60, and to draw upon Leghorn at 48, and Paris Bills are now 25.50, at what rate at least must I be able to draw upon Leghorn to execute this order ?

F.	Paris	Remit	$\dfrac{p}{g}$	$\dfrac{25.50}{25.60}$
S.	Leghorn	Draw	$\dfrac{p}{g}$	$\dfrac{x}{48}$

$$25.60 \times 48 \div 25.50 = 48\frac{3}{16} \text{ Answer.}$$

EXAMPLE 8.

If I have to remit upon Paris at 25.60, and to draw upon Leghorn at 48, and the Leghorn rate is now $48\frac{1}{4}$, to what extent may I limit the Paris rate ?

F.	Paris	Remit	$\dfrac{p}{g}$	$\dfrac{x}{25.60}$
S.	Leghorn	Draw	$\dfrac{p}{g}$	$\dfrac{48\frac{1}{4}}{48}$

$$25.60 \times 48 \div 48\frac{1}{4} = 25.46 \text{ Answer.}$$

EXERCISES.

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Ex. 1. I have an order from Leghorn to remit Bills upon either Paris at 25.60, Amsterdam at 12 2, Hamburg at 13 10, or Genoa at 25.75, but the present rates being, Paris 25.50, Amsterdam 12 0½, Hamburg 13 8½, and Genoa 25.65, which Paper should I select?

2. I have an order from Paris to remit Bills upon either Amsterdam at 12 1½, Hamburg at 13 10¼, Frankfort at 152, Madrid at 38¼, or Leghorn at 48¼—The present rates are on Amsterdam 12 1, Hamburg 13 9½, Frankfort 150½, Madrid 38¾, and Leghorn 48¾, which Paper should I select?

3. I have orders from Hamburg to draw upon either of the following places, or the best that the variations from the following rates may allow; viz. upon Paris at 25.40, Amsterdam at 12 1, Cadiz at 38, or Lisbon at 49¼—Which place should I select, the present rates being as follows; on Paris 25.50, Amsterdam 12 2½, Cadiz 37½, and Lisbon 48½?

4. I have an order to remit Bills upon Amsterdam at 12 1, and to draw upon Hamburg at 13 10, or at equivalent rates; but the present rates being on Amsterdam 12 2½, and Hamburg 13 12, should I execute this order?

5. I have an order to remit Bills upon Leghorn at 48¼, and to draw upon Lisbon at 48½, or at equivalent rates.—Should I execute this order at the present rates of Leghorn 48¾, and Lisbon 49?

6. The rates to which I am limited being, in a compound order, to remit on Leghorn at 48½, and to draw upon Paris at 25.45, or at not less advantageous rates—Am I authorized to act, the present rates being on Leghorn 48¾, and Paris 25.37½?

Ex. 7.   My orders being to remit upon Paris at 25.50, and to draw upon Amsterdam at 12 2, and the price on Paris being 25.40, at what equivalent price should be the rate upon Amsterdam, to enable me to act without loss to my correspondent ?

8.   I have orders to remit upon Paris at 25.55, and to draw upon Hamburg at 13 10 ; the Hamburg rate has improved for drawing to 13 9½, to what depreciation in the Paris rate am I limited ?

9.   I have to remit upon Lisbon at 50, and to draw upon Leghorn at 48½—If the Lisbon rate rises to 50½, to what rate shall I be limited for Leghorn ?

10.   I have to remit upon Lisbon at 50, and to draw upon Madrid at 37—If the Madrid rate improves for drawing to 37½, to what depreciation may be the rate at which I may take Paper upon Lisbon ?

11.   I have to remit upon Lisbon at 50, and to draw upon Paris at 25.50—If the Paris rate improves to 25.37½, to what limit shall I be restrained upon Lisbon ?

12.   I have to remit upon Lisbon at 50, and to draw upon Genoa at 25.60—If the Lisbon rate rises to 50½, at what equivalent rate may I draw upon Genoa ?

13.   If I have to remit upon Paris at 25.65, and to draw upon Lisbon at 50, at what rate may I draw upon Lisbon, if the Paris rate falls to 25.55 ?

ANSWERS.

Ex. 1. Genoa   —   Difference   3.9 per 1000

2. Amsterdam   —   2.1 — 1000

3. Paris   ·   —   4.0 — 1000

4. Rates unfavourable   Ex. 5. Rates equivalent

6. Rates unfavourable   7. Amster. Flor. 12 1.04

8. Paris Frs 25 49.1   9. Leghorn Pence 48.98,5

10. Lisbon Pence 50.6¾   11. Lisbon Pence 50.24

12. Genoa Lire 25.34,6   13. Lisbon Pence 50.19

BULLION OPERATIONS.

Bullion Operations are the import and export of Gold and Silver, either coined or uncoined, to create funds for the purposes of exchange operations.

The chief object of the calculations connected with Foreign Bullion Operations, is to find either from the prices in the two countries concerned, what rate or Par of exchange they yield, or from the price and rate of exchange at one of the places, what arbitrated price is equivalent at the other place.

In the valuation of Gold and Silver in this country, the price of either is determined from the quantity that it contains of metal of a certain purity, called Standard, which standard quantity necessarily depends upon the purity of the given metal, as determined by a process called its Assay.*

In the following consideration of these operations, we shall, first, under the head of the Valuation of Bullion, show the manner in which the fine and standard weights of Bullion are calculated, and the mode in which it is valued, both in this country and in France, the Netherlands, and Germany; and, then, under the head of Arbitrations of Bullion, the most usual comparisons that are made between these values, both for arbitrated Pars and Prices.

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* English Assays are thus made. Correct Trial Plates of Gold and Silver of the respective standard purities, and arbitrary weights, relatively adjusted for each of these metals as described in the following page, being provided, 24 Carats of Gold or 12 Ounces of Silver, are accurately taken both of the metal to be assayed and of its respective Trial Plate. These two pieces are then subjected to the same process of refining. Silver being refined wholly by heat in the assay furnace, but Gold requiring an additional process by chemical solvents in order to separate the Silver with which it is commonly combined. The pure metals, thus obtained from the *assay* and *trial* pieces, are then weighed one against the other, and if the *former* is the *heavier*, it is said to be *better*, and if *lighter, worse*, than the Standard. Thus, if the produce of an assay piece of Gold is heavier than the same of its trial piece by 7-1ths of 1-24th of the original weight, it is reported, Better 1 Carat 3 Grains, which is equivalent to its fineness being 23 Carats 3 Grains.